ANALYSIS OF THE SPONTANEOUS IGNITION OF A TURBULENT GAS JET IN AN OXIDIZER STREAM

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An approximate method is proposed for analyzing the spontaneous ignition of a turbulent gas jet in an oxidizer stream. The results are compared with known test data.

It is becoming common practice to burn gaseous fuel which enters the combustion chamber in the form of a pure jet not premixed with air. The stability of the combustion process under such conditions is most often ensured by special stabilizers or preignition devices. In many cases, however, such devices become superfluous. At high temperatures, for example, the fuel jet may ignite spontaneously in the oxidizer stream and the combustion process will begin here at some distance from the nozzle. The length of that distance is a very important parameter which determines the overall combustion characteristics: at small distances the combustion is effected by the diffusion mechanism, at large distances it becomes a combustion of an almost homogeneous mixture.

Obviously, a solution of this problem in the general formulation calls for a simultaneous analysis of the equations of motion, energy, and turbulence for a turbulent boundary layer with chemical reaction taken into account [1, 2]. Because of the practical difficulties involved in such an analysis, due to our limited knowledge about the kinetics of chemical reactions, it is preferable to explore approximation methods of analysis. One such method will be described here.

Our observations as well as the test data obtained by several other authors [3] have shown that the ignition of turbulent jets is qualitatively similar to the ignition of laminar jets [4]. In both cases the gas ignites within the free layer formed by the jet and the confluent stream, while up to the point where combustion begins the parameters of the boundary layer do not change in any way. This circumstance is helpful in simplifying the problem.

We will consider the process of spontaneous ignition in a fuel jet of finite dimensions in a confluent stream. In order to analyze this phenomenon, we will examine the behavior of a gaseous volume element moving along a stream-line in the boundary layer while its temperature and concentration vary.

We will assume that the chemical reactions (prior to ignition) do not affect the boundary-layer parameters and thus neither the parameters of the inner zone within the given volume, and that the turbulent flow can be described by averaged parameters disregarding the effect of fluctuations on the rate of reactions. As usually, we will also assume a constant pressure in the boundary layer. Let the ignition of our volume element occur at the point along its trajectory where it will remain for a length of time equal to the induction period, i.e., let the ignition occur when the following condition is satisfied:

$$\int_0^{\tau_i} \frac{d\tau}{\tau_i} = 1. \qquad (1)$$

Calculations made for various streamlines within the boundary layer will yield a set of points at which the said condition is satisfied. The geometric locus of these points represents some surface which will be called the nominal ignition boundary. The point on this surface nearest to the nozzle will be called the ignition point. This is the only physically meaningful point on that boundary surface. All other points cannot be identified with the flame surface, because the assumptions under which they have been calculated

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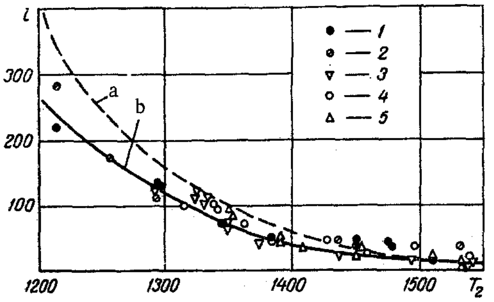

Fig. 1

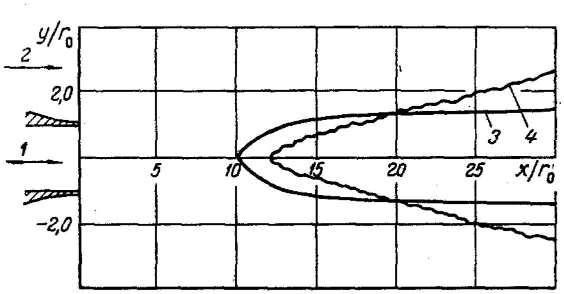

Fig. 2

Fig. 1. Tested and calculated curves of the distance from the ignition point l as a function of the temperature of the hot gas T_2, for various mixture ratios at the same velocity ($u_1 = 105$ m/sec): $\alpha = 0.36$ (1), 0.56 (2), 0.77 (3), 0.94 (4), 1.2 (5). Distance ($1 \cdot 10^{-3}$ m), temperature T_2 (°K). (a) Calculated curve, (b) tested curve.

Fig. 2. Comparison of tested and calculated values for the spontaneous ignition of a hydrogen jet in a supersonic air stream: (1) hydrogen, (2) air, (3) calculated nominal ignition boundary, (4) location of the flame front during the experiment in [3].

do not correspond to the conditions prevailing at the flame front (points on the flame front located along the stream below the ignition point are affected by the combustion process already underway, which is not taken into account in the calculation).

It is difficult to evaluate integral (1) in the general form and the numerical method will be used here. For this purpose, the integral is replaced by a sum whose value will be found by moving along a streamline. We have then

$$\int_0^{\tau_i}\frac{d\tau}{\tau_i}=\int_0^{l}\frac{ds}{V\tau_i}=\sum_{j=0}^{n}\frac{\sqrt{(x_{j+1}-x_j)^2+(y_{j+1}-y_j)^2}}{\sqrt{u^2+v^2}\,\tau_i}, \tag{2}$$

where dS is a streamline element, $x_{j+1} = x_j + \Delta x$ and $y_{j+1} = y_j + (v/u)\Delta x$ are the coordinates of a streamline, and Δx is an increment of the independent variable.

If the density and the longitudinal component of velocity are known, then the transverse component of velocity is determined from the continuity equation:

$$v=-\frac{1}{\rho y^k}\cdot\frac{d}{dx}\int_0^y \rho u y^k dy, \tag{3}$$

where k = 0 or k = 1 for the plane case or the axially symmetric case respectively.

Having determined u and ρ, one can calculate successively all points along the trajectory.

Thus, in order to evaluate the integral, one must determine first all parameters along a streamline within the boundary layer and then how the ignition lag time τ_i varies as a function of the pressure, the temperature, and the concentration.

In the first step of the solution one may proceed by any method used for calculating a free turbulent boundary layer as, for example, the method of the equivalent heat conduction problem [5]. In our study this method will be extended to the mixing of high-speed anisothermal jets with essentially different physical properties.

The gist of this method is that the equations of motion, energy, and diffusion for a boundary layer – written in terms of excess momentum ($\Phi_I = \rho u \Delta u$), total heat content ($\Phi_H = \rho u \Delta i^*$), and concentration ($\Phi_C = \rho u \Delta c$) of a substance – yield the following equation of heat conduction in ξ, y coordinates:

$$\frac{\partial \Phi_i}{\partial \xi_i}=\frac{1}{y^k}\cdot\frac{\partial}{\partial y}\left(y^k\frac{\partial \Phi_i}{\partial y}\right) \tag{4}$$

$$(i = I,\ C,\ H).$$

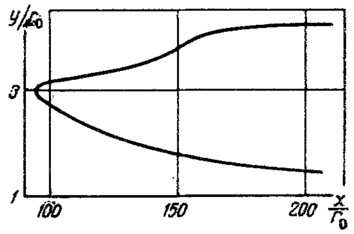

Fig. 3. Nominal ignition boundary for a cold hydrogen jet in a supersonic air stream ($p = 4 \cdot 10^3\ \mathrm{N/m^2}$).

The ξ-coordinate is a transformed coordinate coupled to the longitudinal physical coordinate x through coefficient A, which determines the rate of turbulent exchange in the boundary layer: $\sqrt{\xi_i} = A_i x$. For the dynamic case it has been found [6] that

$$A_{\mathrm{I}} = 0{,}035\left(\frac{\rho_2}{\rho_1}\right)^{0,15}\left(1 - \frac{u_2}{u_1}\sqrt{\frac{\rho_2}{\rho_1}}\right). \tag{5}$$

At the given initial and boundary conditions, Eqs. (4) can be solved and the distributions of excess fluxes can be found in x, y coordinates. These relations, together with the equation of state and the expressions defining the relation between local heat capacity or between molecular weight and concentration ($C_p = \sum_j c_j C_{pj}$, $\mu^{-1} = \sum_j c_j/\mu_j$), apparently suffice for determining the parameters at any point in a boundary layer at given ratios of velocities ($m = u_2/u_1$), of temperatures ($\omega = T_2/T_1$), of molecular weight ($\delta = \mu_2/\mu_1$), and of heat capacities ($\varepsilon = C_{p2}/C_{p1}$), and at a given Mach number ($\beta = (k-1)M_1^2/2$) for the jet.

We show here the final expressions for calculating the dimensionless flux density ($\rho u/\rho_1 u_1$), mass concentration (c/c_1), longitudinal velocity (u/u_1), and temperature (T/T_1) in the case when streams with arbitrary physical properties mix, on the assumption that the parameters of the jet and of the confluent stream remain constant across the nozzle throat,

$$\frac{\rho u}{\rho_1 u_1} = \frac{b}{2} \pm \sqrt{\frac{b^2}{4} + d}, \tag{6}$$

where

$$b = \frac{1}{\varepsilon\omega}\left\{\frac{\Phi_{\mathrm{C}} + (1 - \Phi_{\mathrm{C}})\varepsilon}{1 + \Phi_{\mathrm{C}}(\delta - 1)} m\delta + 2\beta m(1 - m)\Phi_{\mathrm{I}} - (1 + \beta - \varepsilon\omega - \beta m^2)\Phi_{\mathrm{H}}\right\},$$

$$d = \frac{1}{\varepsilon\omega}\left\{\frac{\Phi_{\mathrm{C}} + (1 - \Phi_{\mathrm{C}})\varepsilon}{1 + \Phi_{\mathrm{C}}(\delta - 1)}\Phi_{\mathrm{I}}(1 - m)\delta + \Phi_{\mathrm{I}}^2(1 - m^2)\beta\right\}.$$

Here m, ω, δ, ε, and β are the initial values of the respective parameters in this problem, and Φ_i are the solutions to Eqs. (4) corresponding to a uniform distribution of parameters across the nozzle throat:

$$\Phi_i = \frac{1}{2}\left\{\operatorname{erf}\frac{y + r_0}{2\sqrt{\xi_i}} - \operatorname{erf}\frac{y - r_0}{2\sqrt{\xi_i}}\right\}$$

for the plane case (k = 0) or

$$\Phi_i = \int_0^{r_0} \exp\left(\frac{y^2 + \rho^2}{4\xi_i}\right)\frac{J_0\left(\frac{\rho y}{2\xi_i}\right)}{2\xi_i}\rho d\rho$$

for the axially symmetric case (k = 1). The sign under the square root in (6) is based on physical considerations as, for example, on an obvious estimate of the quantity $\rho u/\rho_1 u_1$ at $\xi \to 0$ or at $\xi \to \infty$;

$$\frac{c}{c_1} = \frac{\Phi_{\mathrm{C}}}{\frac{\rho u}{\rho_1 u_1}}, \tag{7}$$

$$\frac{u}{u_1} = (1 - m)\frac{\Phi_{\mathrm{I}}}{\frac{\rho u}{\rho_1 u_1}} + m, \tag{8}$$

$$\frac{T}{T_1} = \frac{\frac{\Phi_{\mathrm{H}}}{\rho u/\rho_1 u_1}\{1 + \beta - (\varepsilon\omega + \beta m^2)\} + \left\{\varepsilon\omega + \beta m^2 - \beta\left(\frac{u}{u_1}\right)^2\right\}}{\frac{c}{c_1} + \left(1 - \frac{c}{c_1}\right)\varepsilon}. \tag{9}$$

The advantage of using this method for analyzing the ignition criteria is that it renders it possible to calculate a free boundary layer of a jet on a continuous basis, without the necessity of dividing the latter into the initial, the transition, and the main stage (the ignition point may be located in any of these three stages).

For the appropriate functional relation between the ignition lag time and the flow parameters, one consults available published data. Thus, for calculating benzene–air mixtures we use the relation proposed by B. Mullins [7]:

$$\tau_i^* = 1{,}26 \cdot 10^{-8} \exp \frac{46000}{T},$$

while for a hydrogen–air mixture we use the relation proposed by I. Momchiloff et al. [8]:

$$\tau_i^* = \frac{8 \cdot 10^{-6}}{P} \exp \frac{9600}{T}.$$

Inasmuch as these formulas, which are valid for a definite range of mixture ratios, do not contain an expression relating τ_i to the fuel and oxidizer concentrations – which is important in the calculation – it becomes necessary to introduce here an additional correction factor $f(\alpha)$ which is a function of the local excess air α, so as to make $\tau_i = \tau_i^* f(\alpha)$. This factor must ensure the satisfication of the physical requirement that τ_i increase infinitely as the jet boundaries are approached, where the concentration of fuel and oxidizer tends toward zero. One possible expression for $f(\alpha)$ is

$$f(\alpha) = \frac{1}{c_F^{\nu_1} c_0^{\nu_2}},$$

where c_F, c_0 are the concentration of fuel and of oxidizer respectively in a given mixture.

For composite reactions between hydrocarbons and air it has not been possible yet to determine ν_1 and ν_2 reliably enough, because of large discrepancies between their values obtained experimentally by various authors [9, 10]. Some explanation for this discrepancy is found in [11], where the author shows that the values of ν_1 and ν_2 depend largely on the test range of temperatures.

As a first approximation we choose $\nu_1 = \nu_2 = 0.5$ for benzene–air mixtures.

For a thoroughly studied reaction, such as the hydrogen–air reaction, the expression for $f(\alpha)$ can be derived by a direct fitting of test data. Experience has shown [8] that in this case τ_i is only weakly dependent on the mixture ratio, as long as $0.4 \le \alpha \le 2.0$, and that $f(\alpha)$ may be written as $f(\alpha) = \alpha^{-0.09}$ for lean mixtures or $f(\alpha) = \alpha^{-0.2}$ for rich mixtures (since $1/c_F \sim \alpha$ while $1/c_0 \cong 1$ at large values of α).

Let us now analyze the test results pertaining to the ignition of benzene–air jets and of hydrogen jets in air.

In the test performed by M. T. Bortnikov a jet of benzene–air mixture (jet diameter 41 mm, mixture temperature 400°K) was kindled by means of a wide coaxial hot jet (diameter 200 mm). The velocity of this hot jet containing products of complete combustion was maintained the same in all tests ($u_2 = 10$ m/sec). The variables here were the heating gas temperature ($T_2 = 1213$-1540°K), the fuel–air jet velocity ($u_1 = 30$-200 m/sec), and the mixture ratio.

The essential results of these tests are shown in Fig. 1 in terms of the distance from the ignition point l as a function of the heating gas temperature T_2. Calculations were also made for the same conditions. Despite the simplifying assumptions, the calculated curve seems to agree satisfactorily with the test data. The calculations have shown that the ignition point under these particular circumstances lies near the "hot" edge of the boundary layer, since the mixture temperature is still the predominant factor affecting the induction period, even though the latter depends also on the mixture ratio. An analogous situation prevails also at other temperatures of the heating jet.

Calculations were also made for a variable velocity of the fuel–air mixture with the other parameters of both streams held constant ($T_2 = 1430$°K, $T_1 = 400$°K). They have shown that a large increase in the velocity of the cold jet ($u_1 = 30$-200 m/sec) at a constant velocity of the hot stream causes the distance to the ignition point to increase only 2.5 times; the tests have also revealed a still weaker effect of the jet velocity (the distance to the ignition point decreased from 50 to 30 mm).

The ignition tests with a hot hydrogen jet and a confluent supersonic air stream [3] have also established that a subsonic cylindrical jet of hot hydrogen ($T_1^* = 1000$°K, $M_1 = 0.15$) ignites when it merges into a confluent supersonic stream of hot air ($T_2^* = 1300°K$, $M_2 = 1.54$, $P = 10^5\ N/m^2$). At this time, the ignition point lies on the jet axis and the flame front travels at a low angle to the axis.

According to calculations (Fig. 2), the ignition point corresponding to the test conditions lies on the jet axis and the nominal flame boundary moves closer to the jet axis than the developing flame front. The latter has, apparently, to do with the fact that, in calculating the ignition boundary, no account has been taken of the transfer of heat released by the reaction. It becomes evident, if one compares the test data with the calculated values, that the distance to the ignition point can be determined quite accurately.

If the flow conditions are changed and a cold hydrogen jet at the velocity of sound ($T_1^* = 300°K$ and $M_1 = 1.0$) is fed into a hot supersonic stream ($T_2^* = 2000°K$ and $M_2 = 1.5$), then, according to calculations, the ignition point shifts from the axis toward the outer edge of the boundary layer and much farther away from the nozzle throat (Fig. 3). The position of the nominal ignition boundary resembles here the pattern observed earlier with a jet of benzene–air mixture.

One may conclude from this analysis that the parameters which govern the spontaneous ignition of a jet are the temperature and the velocity of the hotter stream (whether fuel or oxidizer), inasmuch as the ignition point lies near that edge of the boundary layer which adjoins the hotter stream.

This analysis is approximate and several factors have been disregarded here which could, under circumstances, be significant. For example, the boundary layer which builds up at solid surfaces of the nozzle could in some cases contribute to ignition. It may be assumed, however, that the method will be useful for analyzing this effect too, since a boundary layer modifies only the initial conditions for Eqs. (4) and, therefore, will cause a change in the function $\Phi(\xi_i, y)$ only.

NOTATION

τ	is the time;
τ_i	is the induction period (lag time), sec;
l	is the distance to the ignition point;
u, v	are the velocity components;
ρ	is the density;
P	is the pressure;
c	is the concentration of fuel;
T^*	is the drag temperature;
T	is the temperature;
$\Delta u = u_1 - u_2$	is the excess velocity;
$\Delta c = c - c_2$	is the excess concentration;
$\Delta i = i^* - i_2^*$	is the excess drag enthalpy;
ξ	is the transformed longitudinal coordinate;
A	is the mixing factor
x, y	are the orthogonal coordinates;
Φ_i	is the solution to the parabolic equations (4);
J_0	is the Bessel function of first kind and zeroth order;
erf	is the Kramp function;
r_0	is the initial radius of the jet;
m, ω, δ, ε	are the ratios of velocities, of temperatures, of molecular weights, and of heat capacities respectively;
α	is the excess air;
M	is the Mach number.

Subscripts:

1 refers to jet;
2 refers to confluent stream.

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